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In 2005 all ECB publications will feature a motif taken from the €50 banknote.

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Abstract

This paper presents a new framework allowing strategic investors to generate yield curve projections contingent on expectations about future macroeconomic scenarios. By consistently linking the shape and location of yield curves to the state of the economy our method generates predictions for the full yield-curve distribution under different assumptions on the future state of the economy. On the technical side, our model represents a regime-switching expansion of Diebold and Li (2003) and hence rests on the Nelson-Siegel functional form set in state-space form. We allow transition probabilities in the regime-switching set-up to depend on observed macroeconomic variables and thus create a link between the macro economy and the shape and location of yield curves and their time-series evolution. The model is successfully applied to US yield curve data covering the period from 1953 to 2004 and encouraging out-of-sample results are obtained, in particular at forecasting horizons longer than 24 months.

Keywords: Regime switching, scenario analysis, yield curve distributions, state space model

JEL classification: C51; C53; E44

Non-technical summary

This paper develops a regime-switching model which can be used to generate long-term yield curve projections for the shape and location of the observable yield curve. The maturity dimension as well as the time-series evolution of yields are incorporated into the modelling framework. In essence the model facilitates conditional yield curve projections to be formed for the whole curve simultaneously while ensuring that the time-series path followed by the curve is linked to macroeconomic variables in a consistent manner. In this way, macroeconomic variables are used as conditioning information for the projections.

It is important to emphasise that it is not the purpose of the model to produce superior yield curve predictions i.e. predictions that in any sense are assumed to out-guess the market and thereby may serve as a basis for tactical investment decisions aimed at outperforming a given benchmark strategy. Rather it is a tool, which supports the investment process related to strategic asset allocation decisions.

In addition to providing a consistent framework for projecting the yield curve an added benefit of the modelling framework is that its output is readily interpretable in an environment where decisions to some extent are based on scenarios: such as it is typically the case in investment committees in public organisations and private investment houses. Furthermore, it enforces direct communication between the strategic and tactical levels of the investment process by linking in an intuitive manner the macro economic scenarios, their estimated occurrence probabilities, and the expected shape and location of yield curves they give rise to.

To estimate the model we rely on the Kalman filter expanded by Hamilton's regime switching methodology. The functional form of the yield curve, in the maturity dimension, is approximated by the three-factor Nelson and Siegel parametric specification, and the time-series evolution of yield curve factors is assumed to follow a regime-switching vector autoregressive model. In this way the observation equation is given by the functional form of the Nelson and Sigel (1987) and the state equation is given by the vector autoregressive structure.

The model is estimated on US data covering the period from 1953 to 2004 and produces promising results both in- and out-of-sample. In-sample, three clearly distinct regime dependent yield curves are identified: one is regularly upward sloping; one is very steeply upward sloping; and one is flat. In a Monte Carlo study we report encouraging out-of-sample results: a comparison of the proposed state-space regime-switching model to the methodology of Diebold and Li (2003) shows that the regime-switching model is significantly better at forecasting horizons longer than 24 months. In an appendix it is also demonstrated, by example, that the model produces the expected interaction between the macro economic variables and the yield curve evolution.

1. Introduction

This paper presents, estimates and forecasts a regime-switching yield-curve factor model where transition probabilities are time-varying and depend on macro economic factors. The methodology relies on a state space formulation of the Nelson-Siegel (1987) parsimonious description of the shape and location of nominal yields. It incorporates regime-switching behaviour in the time-series evolution of the slope factor. As such, the proposed model can be seen as a regime-switching expansion of Diebold and Li (2003).

Our model aims at evolving yields curves over long time horizons. It facilitates generation of history-consistent yield curve scenarios contingent on future paths of a set of macro economic variables. It is important to emphasise that it is not the purpose of the model to produce superior yield curve forecasts i.e. forecasts that in any sense are assumed to out-guess the market and thereby may serve as a basis for tactical investment decisions aimed at outperforming a given benchmark strategy. Rather it is a tool, which supports the investment process related to strategic asset allocation decisions. In addition to providing a consistent framework for projecting the yield curve an added benefit of the model is that its output is readily interpretable in an environment where decisions to some extent are based on scenarios: such as it is typically the case in investment committees in public organisations and private investment houses. Furthermore, it enforces direct communication between the strategic and tactical levels of the investment process by linking in an intuitive manner the macro economic scenarios, their estimated occurrence probabilities, and the expected shape and location of yield curves they give rise to.

An implicit assumption within our modelling framework is that the causality runs from the joint historical evolution of yield curves and macro economic variables to the future path taken by the yield curve. This is in contrast to some of the previous work done on the relation between yields and macro economic factors [see, among others, Estrella and Hardouvelis (1991), Estrella and Mishkin (1996), Fama (1990), Estrella, Rodrigues and Schich (2002), Mishkin (1990) and Estrella and Mishkin (1998)]. In this strand of the literature the causality is assumed to run from the yield curve, in particular from its slope, to the macro economy. Loosely speaking, the argumentation put forth in the above mentioned papers rests on the assumption that agents form expectations about the realisation of the future state of the economy and price assets contingent here upon. Consequently, the yield curve contains information about the future states of the economy, since it is an aggregation of the pricing kernels used by the individual agents of the economy. This allows for the testing of two main hypotheses: one concerns the information contained in the yield spread to predict future inflation and the other concerns the ability of the yield spread to predict future economic activity. The former hypothesis builds on the Fisher decomposition of nominal yields. According to the Fisher decomposition nominal yields are composed of the sum of the expected real interest rate and the inflation rate; hence, yields observed over time for a given maturity contains information about expected inflation measured at the time-horizon covered by that particular maturity. If the term structure of real rates is assumed to be flat and agents of the economy are assumed to be rational then a regression of the time-series differences of observed inflation on yield spreads and a constant should give a significant non-zero regression parameter on the yield curve spread variable. The literature produces mixed results on this relation. In general, the yield spread is not very accurate in predicting short-term inflation but forecasts do get slightly better as the forecasting horizon is increased [see, among others, Mishkin (1990, 1991)]. More encouraging results are found when the latter hypothesis on the relation between the yield curve and real activity is tested. The yield spread is found to be a good predictor for the occurrence of recessions. The economic intuition of this linkage is less straight forward when compared to the previous mentioned relation between the yield spread and inflation, but rests on the expectations hypothesis in conjunction with a monetary policy reaction function and the ability of the central bank to affect economic activity on the longer horizon. An example of the presumed mechanisms is the observation that flat or inverted yield curves tend to precede recessions, as it was the case in the late 1980's and around year 2000. Tests of the hypothesised relation are conducted through regression analysis and the literature generally produces positive evidence [see, for example, Estrella and Hardouvelis (1991) and Estrella et al (2002)].

Another strand of the literature, which is closer to our modelling philosophy, has grown from the affine term-structure models of, for example, Duffie and Kan (1996) and Dai and Singleton (2000), and draws a more direct connection between macro economic variables and the evolution of the term-structure [see also Hordahl, Tristani and Vestin (2002), Piazzesi (2001), Ang and Piazzesi (2003)]. Other papers in the family of affine models have integrated regime-switches in the modelling of yields [see, for example, Ang and Bekaert (2002), Dai, Singleton and Yang (2003), Bansal and Zhou (2002), Driffill and Kenc (2003), Bansal, Tauchen and Zhou (2003), and Evans (2003)]. These models tend to focus on regime-switches in the parameters characterising the mean, the mean-reversion speed and the volatility of the short rate process and generally allow for the presence of two states.

As such, this class of models offers a powerful framework for evolving the yield curve forward conditional on realisations of macro economic variables. However, since the framework rests on the assumption of no-arbitrage and consequently models the yield curve *dynamics* under the risk-neutral measure it is not immediately applicable to yield curve projections under the empirical measure. In particular, as a consequence of the no-arbitrage restriction the evolution of yields under the risk-neutral measure are drift less. To provide a mapping between the risk-neutral measure and the empirical measure (to facilitate estimation of the models, and, so to speak, bridge the wedge that would otherwise arise between the drift less risk neutral measure and the "drifting" empirical measure) a certain functional form on the Radon-Nikodym derivative (risk premium) is imposed. This provides for a translation between the measures and adds additional constraints on the modelling framework and the model specification. A key issue here is the uniqueness of the pricing measure, which rests on the assumption about market completeness. If markets are incomplete there does not exist a unique pricing measure and thus no single way to specify the functional form of the market risk premium. While this ambiguity does not cause major problems when the focus of attention is on relative pricing at a given point in time, it is problematic when addressing the issue of long-term evolution of yield curves: for longer horizons the drift term will dominate the volatility term in the underlying diffusion process.[3] In effect, the choice of measure under which the modelling is conducted, is intimately related to the purpose of the model set forth, the number of assumptions the econometrician is willing to make, and the context in which the model should be used. While our discussion of the "arbitrage-free" framework should not be seen as a criticism or an attempt to question the applicability of these models to the issue at hand, it does highlight the central differences between modelling under the risk-neutral and empirical measures in the context of long-run forecasting of yields. It is an empirical question whether the additional structure that is implied by the risk neutral framework improves or exacerbates the forecasting performance of yield curve models when applied to longer forecasting horizons.

Our modelling framework differs in several important respects from those described above: it integrates a three-state regime-switching model for the yield curve under the empirical measure, it evolves the yield curve dynamically for all maturities at the same time, and it allows macro economic factors to influence the transition probabilities. The focus is immediately on the variable of relevance i.e. the nominal yield under the empirical measure and we do not have to resort to assumptions about the functional form and time-series evolution of the market price of risk. Additionally, from an implementation view point, the estimation method applied in our modelling framework avoids the involved two-step maximum likelihood scheme used by Ang and Piaezzesi (2003) and the need for calibrating a joint macro-model and yield curve model as in Hordahl et al (2002).

The applied estimation technique relies on Hamilton (1994) and Kim and Nelson (1999). When applied to a sample of monthly US yields covering the period from 1953 to 2004 we estimate three clearly distinct regime curves: one is regularly upward sloping; one is very steeply upward sloping; and one is flat. The model fits data well in sample. Since our main objective is to evolve the yield curve forward it is by definition assumed that causality runs from the macro economy to the yield curve. In particular, we argue that the long end of the yield curve is relative stable and that a central bank in its efforts to guide the economy may lower the short-term interest rate (which will increase the yield spread) to counter recessionary pressures, and increase short term interest rates (which will decrease the yield spread) to counter inflationary pressures.

Our modelling framework is designed to aid the process of predicting yield curve evolutions for the longer horizons. In this vein, we report encouraging results from a Monte Carlo study: an out-of-sample comparison to the methodology of Diebold and Li (2003) shows that the regime-switching model is significantly better at forecasting horizon above 24 months. In an appendix we demonstrate, by example, that the model produces the expected interaction between the macro economic variables and the yield curve evolution.

[3] Similar argumentations are put forth be Rebonato et. al (2005). Also Diebold and Li (2003) and Diebold, Rudebusch and Auroba (2003) model directly under the empirical measure.

The rest of the paper is organised as follows. Section two presented the model and the estimation technique. Section three describes the data, the estimation and out-of-sample forecasting results. Section four concludes, and Annex 1 contains a case study on yield curve prediction.

2. The Modelling framework

This section presents the model, how it is estimated and how it can be used to project yield curves for the longer time-horizons.

2.1 The model and estimation technique

The vector of yields *Y* observed at time *t* for different maturities $\tau = (\tau_1, \tau_2, \ldots, \tau_n)$ can be expressed as a function of yield curve factors and yield curve factor sensitivities according to the Nelson-Siegel (1987) parametric description of the shape and location of the yield curve. In our setup we allow for regime-switching behaviour to occur in the factors' mean. By applying an expansion of the general Kalman filter as suggested by Kim and Nelson (1999) it is shown how the likelihood function is constructed. This technique relies on an iterative procedure where the Hamilton filter, i.e. the procedure used to estimate regime-switching part of model, is embedded within the Kalman filter. A key element in the approach is to ensure that the dimension of the Kalman filter stays tractable: hence, at each *i*-iteration the parameter exhibiting regime-switching behaviour is updated through a weighting scheme, where the used weights are determined by the Hamilton filter.

We formulate the model in state-space form using as observation equation:

$$Y_t = H\beta_t^j + e_t, \qquad [1]$$

where *Y* is the vector of yield observations at time *t*, β is the vector of Nelson-Siegel factors, *j* indicate regime affiliation $j \in \{\text{(N)ormal, (S)teep, (I)nverse}\}$, and H is the matrix of Nelson-Siegel sensitivities, i.e.

$$H = \begin{bmatrix} 1 & \frac{1-\exp(-\lambda\tau_1)}{\lambda\tau_1} & \frac{1-\exp(-\lambda\tau_1)}{\lambda\tau_1} - \exp(-\lambda\tau_1) \\ 1 & \frac{1-\exp(-\lambda\tau_2)}{\lambda\tau_2} & \frac{1-\exp(-\lambda\tau_2)}{\lambda\tau_2} - \exp(-\lambda\tau_2) \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-\exp(-\lambda\tau_n)}{\lambda\tau_n} & \frac{1-\exp(-\lambda\tau_n)}{\lambda\tau_n} - \exp(-\lambda\tau_n) \end{bmatrix}, \qquad [2]$$

and *e* is the error-term. It is assumed that $e \sim N(0, R)$ where $R = \sigma_e^2 I$, and *I* is the identity matrix.

To describe the evolution over time of the Nelson-Siegel factors the following state equation is used:

$$\beta_{t|t-1}^j = m^j + F\beta_{t-1|t-1} + v_t, \qquad [3]$$

where $m^j = \left[c_1, c_2^j, c_3\right]'$ is the vector of mean parameters. The matrix F collects autoregressive parameters,

$$F = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_2 \end{bmatrix},$$

v is the error-term and it is assumed that $v \sim N(0, Q)$ and $Q = \sigma_v^2 I$, where I is the identity matrix. $\beta_{t-1|t-1}$ is the probability weighted average of the betas from the i'th Kalman-filter iteration ($\beta_{t-1|t-1}$ contains the values that are used to initialise the Kalman filter when i=1).

The specification of the observation and state equations in [1] and [3] rests on the principles of parsimony, practical applicability and economic theory. The measurement equation, as specified by the Nelson-Siegel function form, is chosen on grounds of parsimony. By using only four parameters at any given point in time it is known to capture the major part of the variability of yields and represent well yield curve shapes relevant for macro economic analysis. The interpretation of the yield curve factors are: the first factor proxies the yield curve level, i.e. the yield at infinite maturity; the second factor can be interpreted as the negative of the yield curve slope, i.e. the difference between the short and the long ends of the curve; the last yield curve factor can be interpreted as the curvature. The fourth parameter, λ, determines the time-decay in the maturity spectrum of each factor, as illustrated in [2].

Practical applicability and economic theory have been guiding the choice of specification for the state equation. Our main purpose is to capture generic yield curve shapes and to link them to the state of the macro economy. The Taylor rule [Taylor (1993)] provides a useful framework for this. In periods of economic downturn, i.e. low GDP growth, the central bank will try to stimulate the economy by lowering short term interest rates (whereby the slope of the yield curve will increase); in periods of high inflation the central bank will try to dampen economic activity by increasing the policy rate (whereby the yield curve will flatten or even become inverse); in all other cases the central bank will make only marginal changes to the short rate and the yield curve will be normally upward sloping. A premise for this rationale is that the long-term rate is relatively constant, which finds support in the Fisher decomposition of nominal yields. Accordingly, nominal yields are composed of the sum of the expected real interest rate and the inflation rate, which in the long run would be stable since the real rate equates the growth of the economy.

In the state equation, the dynamics for the first and third yield curve factors follow AR(1) processes, which finds support in empirical data [see e.g. Diebold and Li (2003)]. To capture the changes in the yield curve slope, as suggested by the combined effects of the Taylor rule and the Fisher decomposition, we propose a three state regime-switching model for the second yield curve factor. Regime-switching is presumed to occur in the mean only.[4] These considerations are reflected in the specification of [3].

As a consequence of this model set up, the prediction errors η are regime-dependant:

$$\eta^j_{t|t-1} = Y_t - H\beta^j_{t|t-1} . \qquad [4]$$

However, since the mean specification of the state equation is capturing the regime-switching behaviour of the system, the variance terms of the Kalman-filter are unaffected by regime-switches. For this reason the conditional prediction-error variance can be expressed as:

$$f_{t|t-1} = E\left[\eta^2_{t|t-1}\right] = HP_{t|t-1}H' + R , \qquad [5]$$

where $P_{t|t-1}$ is the one-step ahead predictor of the variance of β, which is

$$P_{t|t-1} = FP_{t-1|t-1}F' + Q . \qquad [6]$$

The density for each of the j-regimes can then be calculated by

$$l^j_t(\theta) \propto \left| f_{t|t-1} \right|^{-0.5} \exp\left\{ -\frac{1}{2}\eta^j_{t|t-1} f^{-1}_{t|t-1} \eta^j_{t|t-1} \right\} . \qquad [7]$$

To complete the Kalman filter beta as well as it variance needs to be updated. While the updating of the variance proceeds according to the regular Kalman filter, updating beta has to take into account the regime-switching behaviour as given in [3].

Regime-switching probabilities π, in the Hamilton filter, are calculated as

$$\pi_{t|t} = \frac{\pi_{t|t-1}\langle\bullet\rangle D_t}{1'\left(\pi_{t|t-1}\langle\bullet\rangle D_t\right)} , \qquad [8]$$

where $\langle\bullet\rangle$ is the element-by-element multiplication, *D* collects the densities from [7] in a row vector, and **1** is a vector of ones of dimension *j*. According to the Hamilton filter these probabilities are predicted one-step ahead in the following way:

$$\pi_{t+1|t} = p^{Z_t}\pi_{t|t} , \qquad [9]$$

with p^{Z_t} being the transition probability matrix. We expand the regular Hamilton filter by allowing *p* to depend on the realisation of macro economic factors. In particular,

[4] Alternatively, the transition between states could be modelled via an autoregressive model for the slope factor. However, the speed at which this transition is believed to occur would make it difficult to capture when data of quarterly or monthly observation frequency are used.

$$Z_t = \begin{cases} 1 & otherwise \\ 2 & if \qquad \Delta \mathrm{gdp}_t < \Delta \mathrm{gdp}^* \text{ and } \Delta \mathrm{cpi}_t < \Delta \mathrm{cpi}^* \\ 3 & if \qquad \Delta \mathrm{gdp}_t > \Delta \mathrm{gdp}^* \text{ and } \Delta \mathrm{cpi}_t > \Delta \mathrm{cpi}^* \end{cases}, \qquad [10]$$

where $\Delta gdp*$ and $\Delta cpi*$ represent threshold values for the use of alternate transition matrices, as illustrated in [9] and [10].

The updating equations of the expanded Kalman filter then take the following form:

$$P_{t|t} = P_{t|t-1} - P_{t|t-1}H' f_{t|t-1} HP_{t|t-1}, \qquad [11]$$

$$\beta^j_{t|t} = \beta^j_{t|t-1} + P_{t|t-1}H' f^{-1}_{t|t-1}\eta^j_{t|t-1}. \qquad [12]$$

However, to mitigate that the dimension of the Kalman filter grows by j at each iteration (i.e. betas to keep track of would otherwise grow by the number regimes j to the power of the number of iterations), $\beta^j_{t|t}$ has to be "reduced" to just $\beta_{t|t}$. This is obtained by calculating the weighted average of $\beta^j_{t|t}$ using,

$$\beta_{t|t} = B_{t|t}\pi_{t|t} \qquad [13]$$

where $B_{t|t} = \left\lfloor \beta^S_{t|t}, \beta^I_{t|t}, \beta^N_{t|t} \right\rfloor$. Hence, after each iteration the regime-dependent betas are "collapsed" into one single vector.

The likelihood function to be maximised is actually the denominator of [8], i.e. the log of the weighted sum of the densities for each of the regimes:

$$LogL(\theta) \propto \sum_{t=1}^{T} \log\left[1'\left(\pi_{t|t-1}\langle\bullet\rangle D_t\right)\right]. \qquad [14]$$

2.2 Projecting yield curves for the longer time-horizon

Once parameter estimates are obtained the modelling framework described above can be used to evolve yield curves forward conditional on assumed path for the macro economic variables. Scenarios for the yield curve and yield curve distributions can thus be constructed following the steps outlined below.

Step 1: Generate values for the macro economic variables for the desired projection horizon *S*.

Step 2: Calculate the state probabilities $\pi_{t|t}$ using [9] and [10] for $t = (1, 2, \ldots S)$.

Step 3: Use the result from step 2 to calculate $m^j_t = \left[\pi_{t|t}'\hat{\beta}^j_1 \middle| \pi_{t|t}'\hat{\beta}^j_2 \middle| \pi_{t|t}'\hat{\beta}^j_3\right] \; \forall\, t$, which gives an *Sx3* matrix of mean values consistent with the presumed evolution for macro economic variables.

Step 4: Use $\hat{F}$ and the result from step 3 to calculate $E\left[\beta^j_t\right]$ following [3].

Step 5: Insert the result from step 4 into [1] together with $\hat{H}$ to obtain the yield curve predictions for $\tilde{Y}_t(\tau)$ for $t=(1,2,\ldots,S)$ and $\tau=(\tau_1,\tau_2,\ldots,\tau_N)$.

Following these five steps and by varying the values for the macro economic factors, we can produce yield curve evolutions and yield curve distributions under the empirical measure at the desired forecasting horizon.

3. Estimating the model

3.1 Data and estimation

We estimate the model for the US Government money and bond market using nominal yield curve data calculated by the Treasury Department and reported by the Federal Reserve for constant maturities of 3, 6, 12, 24, 36, 60, 84, 120 months.[5] The data cover the period from 1953:4 to 2004:4 and is collected at a monthly frequency. A monthly sampling frequency is also applied to GDP and inflation data. Since GDP data is available at a quarterly frequency it is assumed that months within each quarter have equal GDP figures. The macroeconomic data is calculated as year-on-year percentage changes. Even though this data transformation induces a moving average structure of order eleven in the data, this does not affect the theoretical specification of the model. In fact, the time series properties of the macro factors enter the likelihood function only through the mean of the series; since an induced moving average process does not effect the unconditional mean of the time series no change is required[6]. Figure 1 shows a time series plot of the data.

A normalisation of the data is performed to ensure that yield spreads are comparable across periods where yield levels vary between high and low values. Our data sample spans 1953 to 2004 and the yield curve level varies substantially during this period: e.g. in the beginning of the 1980'ies the 10 year segment of the curve was above 12% while in the 1950'ies and 2001-2004 it was around 2%-4%. This variety naturally affects the possible values that the yield spread can assume. And, since our model classifies regimes according to the value of the yield spread (see [3]) it is natural to rely on the relative size of the

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[5] To ensure a full data history covering 1953 to 2004 we use interpolated data for maturities 6, 24, 84 months, from 1953 to 1958 for the 6 months segment, from 1953 to 1976 for the 24 months segment and from 1953 to 1969 for the 84 months segment.

[6] The macro factors enter as a step-function in the determination of the transition probabilities. Subjective cut-off values for GDP and CPI growth determine the steps in the functions and hence the transition probabilities, in conjunction, of course, with yield curve data. It could be argued that an induced autoregressive process would require a change in the model set up since such a process would affect the unconditional mean of the macro series. However, the steps in the transition probability functions are determined subjectively and the required change is as such easy to implement.

yield curve slope, rather than its absolute value.[7] To retain the dynamic evolution of the yield curve level (i.e. the first Nelson-Siegel factor) we choose to centre the normalisation around the 10 year segment of the curve. More specifically, the following normalisation is implemented:

$$\tilde{Y}_t(\tau_i) = Y_t(\tau_N) - \frac{Y_t(\tau_N) - Y_t(\tau_i)}{Y_t(\tau_N)} \quad \forall i \in \{1, 2, \ldots, N\}, \qquad [15]$$

where N is the highest maturity in the sample.

Cut-off values for the GDP and CPI growth relevant for [10] are chosen at 1% and 4%, respectively.

Figure 2 shows the estimated Nelson-Siegel yield curve factors and the regime-classifications. Parameter estimates are shown in Appendix 2. The model identifies three clearly distinguished regimes with regime probabilities for the active state being above 90% in general and typically very close to unity (see the lower panel in Figure 2). Such a result indicates that data supports the model since values for the densities governing each regime are markedly different in magnitude. A high degree of persistence within each of the identified regimes is also observed (this can be confirmed by visual inspection of the lower panel in Figure 2 and the diagonal elements of the transition matrices in Table 2). From an economic viewpoint this is reassuring because it shows that the yield curve shapes are relatively stable (within given ranges) for a prolonged period of time for each of the identified regimes; this fits well with the general notion of how the business cycle evolves over time. Almost all estimated parameters are significantly different from zero at a 95% level of confidence, judged by QML standard errors. The economic interpretation of the regime-switching slope constant confirms our hypothesis: in the first regime the slope is upward sloping (corresponding to the value of c_2^1); in the second regime the slope is steeply upward sloping (corresponding to the value of c_2^2); in the third regime the slope is nearly flat (corresponding to the value of c_2^3).

3.2 Model evaluation

As also stated in the introduction, the main purpose of the model presented above is to provide a tool, which allows for generation of history consistent long-term yield projections relevant for strategic asset allocation decisions. It aims at facilitating the discussion in investment committees where people of different professional backgrounds interact with the purpose of deciding long-term investment strategies. This is done by providing a methodology allowing for the generation of scenarios for future yield

[7] A more extreme example of this occurs when the model is applied to Japanese data. Yield level in this market is at the time of writing around 1.3% while the short end is around 0%. In our modelling framework, using non-normalised data, such a curve would probably be classified as "normally" upward sloping, which, given the historic evolution of yields, would not be well supported. The normalisation would effective alter the relative slope for the rest of the data material so that similar (absolute) slope at a higher yield levels would be "scaled down": e.g. a 1.3% slope at the level of 6% would result is a normalised short rate level at 6-1.3/6=5.8, which give a normalised slope of 0.2, whereas the normalised slope would be 1.0 at the lower 1.3% level. Regime-classifications would change accordingly.

evolutions based on a set of measurable input parameters. In this way the model provides a common language for traders, economists and senior management.

However, regardless of the development-purpose of the regime-switching model it is still interesting to analyse how well it performs in an out-of-sample forecasting exercise. To serve as a benchmark the Diebold and Li (2003) model is chosen: this is the natural benchmark since the regime-switching model expands the Diebold and Li modelling philosophy. Also, Diebold and Li demonstrate that their model performs well in an out-of-sample forecasting exercise, in particular at the 12 months forecasting horizon.

To explore the relative forecasting performance of the Diebold and Li and the regime switching models a Monte Carlo experiment is setup in the following way:

1. The Diebold and Li and the regime-switching models are estimated on the historical sample.
2. By the use of the block-bootstrapping technique [see e.g. Efron (1979)] a sample of 60 months of data is (re)generated.[8]
3. Using the parameters obtained from step 1, predictions are made for each model 60 months ahead.
4. Prediction errors are calculated for each method against the bootstrapped data as $\varepsilon_{m,\tau,f} = \hat{Y}_{m,\tau,f} - Y_{\tau,f}$, where "m" refers to the method i.e. either Diebold and Li (DL) or the regime-switching (RS) models, "τ" is the maturity of the instrument under consideration i.e. $\tau = \{3,6,12,24,36,60,84,120\}$ months, and "f" is the forecasting horizon in months i.e. $f = \{1,2,...,60\}$.
5. Steps 2 through 4 are repeated 500 times, generating 500 realizations of $\varepsilon_{m,\tau,f,b}$, where the "b" counts the bootstrap sample i.e. $b = \{1,2,...,B\}$, and *B=500*.
6. The root mean squared error for each method, each maturity "τ", and each forecasting horizon "f" across the bootstrapped data paths, is calculated as: $rmse_{m,\tau,f} = \sqrt{1/B\sum_{b=1}^{B}\varepsilon_{m,\tau,f}^2}$.
7. To compare the performance of the two models the Diebold-Mariano technique is used to calculate standard errors on the difference between $rmse_{RS,\tau,f}$ and $rmse_{NS,\tau,f}$ [see Diebold and Mariano (1995)].

Figure 3 below shows the results of the Monte Carlo experiment. One sub-plot is constructed for each of the analysed yield curve segments and the difference between rmse of the regime-switching model and the Diebold and Li model is shown by the lines. To assess the significance of the differences 95% confidence intervals are calculated on the basis on Diebold and Mariano (1995), represented by dotted lines.

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[8] For the Monte Carlo experiment a perfect correlation is assumed between macro economic variables and the yield curve classification obtained on the historical data, to map as closely as possible the scenario-generation intention of the model.

Since the results in Figure 3 are presented as the root mean squared error of the regime-switching model minus the root mean squared error of the Diebold and Li model, a negative number means that the regime-switching model is more precise than the Diebold and Li model, while a positive number means that the regime-switching model is less precise than the Diebold and Li model. Table 3 shows a representative sample of the root mean squared errors of the individual model. From Figure 3 and Table 3 it can be seen that for maturities and forecast horizons below 12 months the Diebold and Li methodology is more precise than the regime-switching model, although the difference is not significant at a 95% level of confidence. This picture changes when a forecasting horizon above 24 months is considered; for the medium to long-term horizon the regime-switching model is forecasting significantly better than the Diebold and Li method, judged by Diebold-Mariano confidence levels.

4. Conclusions

The generation of long-term expectations to the level, shape and evolution of the yield curve are key inputs to the strategic investment process applied by investment managers in private or public organisations alike. Such information is used in the formation of return and risk scenarios for asset classes such as bonds, equities and real estate. One of the challenges in generating yield curve scenarios is to consistently link projections for macro economic variables to the evolution of the yield curve.

In this paper we present a framework that can aid the process by consistently linking expectations on future key macroeconomic variables to the shape and location of the yield curve. Our model can be seen as a regime-switching expansion of the approach taken by Diebold and Li (2003). It captures the evolution of yields in the time-series dimension as well as in the maturity dimension. This is obtained by relying on the Nelson and Siegel (1987) parametric specification of the shape and location of yields and by allowing for regime shifts in the time-series evolution of the yield curve factors. In particular, our model incorporates regime shifts in the yield curve slope, and the regime-transition matrix governing the migration within and between the regimes, is depends on macroeconomic variables.

The model is formulated in state-space form and we demonstrate how to estimate and forecast the model using US nominal yield data observed at a monthly frequency and covering the period from 1953 to 2004. On the historical sample we identify three clearly distinguished yield curve shapes: one regularly upward sloping, one steeply upward sloping curve and finally a flat curve. Out-of-sample results show that the regime-switching model provides additional forecasting power over the Diebold and Li model: at horizons above 24 months the regime-switching model significantly outperforms the non-regime switching alternative.

References

Ang, A. and G. Bekaert, 2002, Regime Switches in Interest Rates, *Journal of Business & Economic Statistics*, April, Vol. 20, No.2, pp 163-182

Ang, A. and M. Piazzesi, A No-Arbitrage Vector Autoregression of the Therm Structure Dynamics with Macroeconomic and Latent Variables, 2003, *Journal of Monetary Economics*, Vol.50, pp 745-787

Bansal, R. and H. Zhou, 2002, Term Structure of Interest Rates with Regime Shifts, *Journal of Finance*, Vol. LVII, No.5, pp 1997-2043

Bansal, R., G. Tauchen, and H. Zhou, 2003, Regime-Shifts, Risk Premiums in the Term Structure, and the Business Cycle, Working paper Duke University

Dai, Q. and K.J. Singleton, 2000, Specification Analysis of Affine Term Structure Models, *Journal of Finance*, Vol. LV, No. 5

Dai, Q., K.J. Singleton, and W. Yang, 2003, Regime Shifts in a Dynamic Term Structure Model of U.S. Treasury Bond Yields, *Working paper Stanford University*

Diebold, F.X. and C. Li, 2003, Forecasting the Term Structure of Government Bond Yields, *Journal of Econometrics, forthcoming*

Diebold, F.X. and R.S. Mariano, 1995, Comparing Predictive Accuracy, *Journal of Business & Economic Statistics*, Vol. 13, No. 3

Diebold, F.X., G.D. Rudebusch, and S.B. Aruoba, 2003, The Macroeconomy and the Yield Curve: A Nonstructural Analysis, *Journal of Econometrics, forthcoming*

Driffill, J., T. Kenc and M. Sola, 2003, An Empirical Examination of Term Structure Models with Regime Shifts, *Working paper Birkbeck College, University of London*

Duffie, D. and R. Kan, 1996, A Yield-Factor Model of Interest Rates, Mathematical Finance, Vol. 6, No.4, pp. 379-406

Efron, B., 1979, Bootstrap Methods: Another look at the Jackknife, Annals of Statistics, Vol. 7, N0. 1, pp. 1-26.

Estrella, A. and F.S. Miskin, 1996, The Yield Curve as a predictor of U.S. recessions, Federal Reserve Bank of New York

Estrella A. and F.S. Mishkin, 1998, Predicting U.S. recessions: financial variables as leading indicators, The Review of Economics and Statistics, Vol.80 pp 45-61

Estrella, A. and Hardouvelis, G.A., 1991, The Term Structure as a Predictor of Real Economic Activity, Vol. 46, No. 2, pp.555-576

Estrella, A., A.P. Rodrigues and S. Schich, 2002, How stable is the predictive power of the Yield Curve? Evidence from Germany and the United States, *Working paper Federal Reserve Bank of New York*

Evans, M.D.D., 2003, Real Risk, Inflation Risk, and the Term Structure, *The Economic Journal*, Vol. 113 (April), pp 345-389

Fama, E.F., 1990, Term-structure forecasts of interest rates, inflation, and real returns, *Journal of Monetary Economics*, Vol. 25, pp. 59-76

Hamilton, J.D., 1994, Time Series Analysis, Princeton University Press

Hördahl, P., O. Tristani and D. Vestin, 2002, A joint econometric model of macroeconomic and term structure dynamics, *ECB working paper*

Kim, C.J. and C.R. Nelson, 2000, State Space Models with Regime Switching, MIT press

Mishkin, F.S., 1990, What does the yield curve tell us about future inflation?, *Journal of Monetary Economics*, Vol. 25, pp. 77-95

Mishkin, F.S., 1991, A multi-country study of the information in the shorter maturity term structure about future inflation, Journal of International Money and Finance, Vol. 10, pp. 2-22

Nelson, C.R. and A.F. Siegel, 1987, Parsimonious Modeling of Yield Curves, *Journal of Business*, Vol. 60, No 4, pp 473-489

Piazzesi, M., 2001, An Econometric Model for the Yield Curve with Macroeconomic Jump Effects, *NBER working paper*, No 8246

Rebonato, R. S. Mahal, M. Joshi, L.D. Bucholz and K. Nyholm, 2005, Evolving Yield Curves in the Real-World Measure: a Simi-Parametric Approach, *Journal of Risk (forthcoming)*

Taylor, J.B., 1993, Discretion version policy rules in practice, Carnegie-Rochester Conference Series on Public Policy, Vol. 39, pp 195-214

ANNEX 1: CASE STUDY ON YIELD CURVE PROJECTIONS

Based on assumed parameter values this section shows how the modelling framework can by used to generate yield curve projections. To illustrate this technique, two alternative hypothetical macroeconomic scenarios are analysed. Yield curves are projected over a horizon of 60 months using 10,000 simulation runs to establish the distribution of future yields. The crux of the modelling framework is, as discussed above, that yield curve projections are made contingent on macro-economic scenarios. Figure 4 shows the two alternative hypothetical scenarios for GDP and inflation used in this section. These scenarios are chosen on an ad-hoc basis to exemplify future normal and pessimistic economic environments. In the normal environment annual GDP growth rates are assumed to gradually decrease from 4.3% to 2.6% while inflation goes up by 1.3% to 2.8% at the end of the horizon. The hypothetical recession scenario assumes that average GDP growth will gradually fall from 3% to -1% and inflation will decrease by 1.5% to 2% at the end of the projection horizon. Furthermore we assume a standard deviation of 1.5% and 1% for GDP growth rate and inflation rates respectively.

The upper two panels in Figure 5 show the average evolution of state probabilities across the conducted simulations. For the hypothetical main economic scenario the probability of a normal curve increases from 3.9% to a maximum of 86.0% percent after 37 months. From here on, the probability decreases slightly to 81.9% at the end of the forecasting horizon. At the same time the probability of a steep curve goes down to 14.7% and the probability of a flat curve increases to 3.4% percent. Due to higher initial inflation rates, the hypothetical pessimistic scenario exhibits a much faster increase in the probabilities of a normal curve: after 10 months the probability of a normal yield curve reaches a maximum of 88.3% and then decreases to 3.6% an the end of the horizon. The lower panels of Figure 5 show the evolution of the average yield curves, where, again the averages are calculated across the conducted simulations. In the hypothetical normal economic scenario, the 3-month yield increases to 4.6% at the end of the 60 months forecasting period, while the 10-year yield reaches 6.1% percent. A completely different yield curve evolution is produced by the hypothetical pessimistic economic outlook. Here yields initially increase until the point where GDP growth decreases. After this point the "generic" steep yield curve state sets in and leads to decreasing yields in the short as well as the long end of the maturity spectrum. Comparing both scenarios, the initial strong GDP growths in the hypothetically main economic scenario has comparably less effect on the location and shape of the yield curve than the initially high inflation rates in the pessimistic scenario. However negative GDP growths rates during the last three years of the hypothetical pessimistic scenario have a major impact on the yield curve shape and location, as is evident from the lower right panel of Figure 5.

Figure 6 shows the yield distributions over the 60 months forecasting horizon. The average patterns depicted in the lower panels of Figure 5 are naturally reflected here, however, in addition the empirical percentiles are shown to give a better feeling on the properties of the model forecasts.

ANNEX 2: PARAMETER ESTIMATES

Table 1: Estimated transition matrices

Main economic scenario p^1	Normal	Steep	Inverse	Recession p^2	Normal	Steep	Inverse	Inflation p^3	Normal	Steep	Inverse
Normal	0.97(*)	0.03(*)	0.05	Normal	0.80(*)	0.00	0.19	Normal	0.96(*)	0.40(*)	0.00
Steep	0.0000	0.97(*)	0.00	Steep	0.17(*)	0.95(*)	0.02	Steep	0.0	0.60(*)	0.00
Inverse	0.03(*)	0.0000	0.95(*)	Inverse	0.03	0.00	0.79(*)	Inverse	0.04	0.00	1.00

Note: Parameter estimates obtained from maximizing [14]. QML standard errors are used to assess the significance of the parameter estimates: (*) indicates that a parameter is different from zero at a 5% level of significance.

Table 2: Estimated parameters

a_1	0.89(*)
a_3	0.84(*)
σ_e	0.01(*)
σ_v	0.20(*)
λ	0.08(*)
c_1	0.04(*)
c_2^1	-0.34(*)
c_2^2	-0.73(*)
c_2^3	-0.07(*)
c_3	0.00

Note: Parameter estimates obtained from maximizing [14]. QML standard errors are used to assess the significance of the parameter estimates: (*) indicates that a parameter is different from zero at a 5% level of significance. Shown parameter estimates refer to the scaled data; rescaling can be done by multiplying by the average value for the level factor in [1] i.e. $mean(\beta_{1,t})$ for $t = \{1, 2, ..., T\}$.

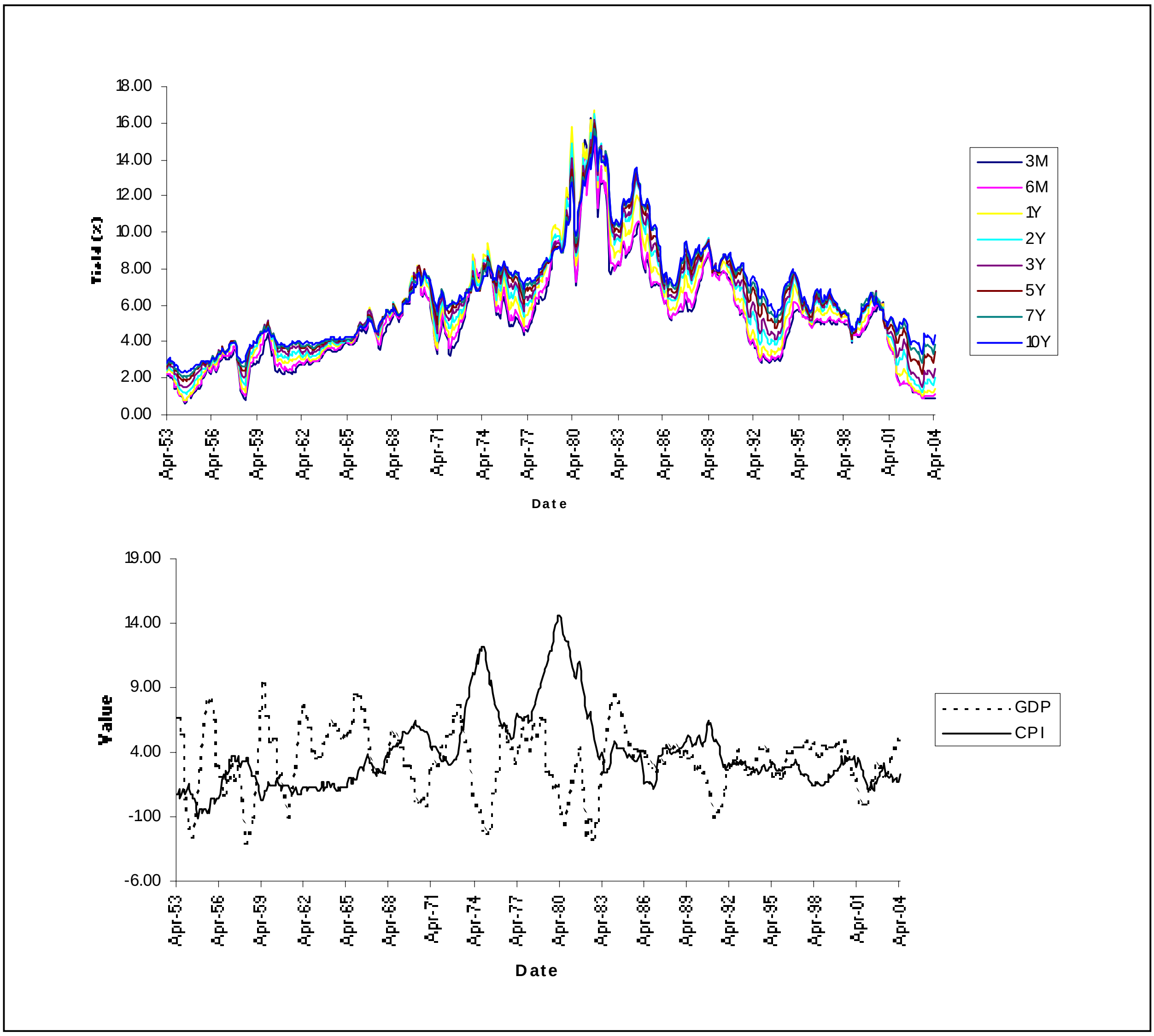
Table 3: Root mean square errors for the models in Basis Points

	RMSE(Regime-Switching model) [basis points]					RMSE(Diebold and Li model) [basis points]				
Horizon\Maturity	3M	6M	1Y	5Y	10Y	3M	6M	1Y	5Y	10Y
1 months	31.85	31.64	31.86	29.71	29.03	31.48	31.65	32.22	31.24	30.71
6 months	87.44	87.73	88.53	84.41	82.49	87.03	87.42	88.36	84.89	83.11
12 months	123.00	123.41	124.49	120.22	118.35	123.62	124.01	125.09	120.93	119.06
24 months	156.72	156.86	158.00	153.90	152.19	160.76	160.59	161.20	155.68	153.56
36 months	174.21	174.28	175.69	172.55	171.13	181.14	180.77	181.36	175.66	173.43
48 months	194.86	195.09	196.68	194.67	193.46	204.18	204.01	204.71	199.33	196.96
60 months	213.82	214.19	215.79	213.93	212.64	225.20	225.21	225.96	220.29	217.59

Note: Root mean squared errors for the regime-switching model and the Diebold and Li model generated on the basis of a Monte Carlos experiment. 500 samples are (re)generated for the maturities covered by the original sample and having a length of 60 monthly observations. For each sample forecasts are performed for both models. The RMSE for each model, for each of the yield curve segments (defined by their modified duration), and for each forecasting horizons from 1 to 60 months are calculated. A representative sample of these results are presented in the table.

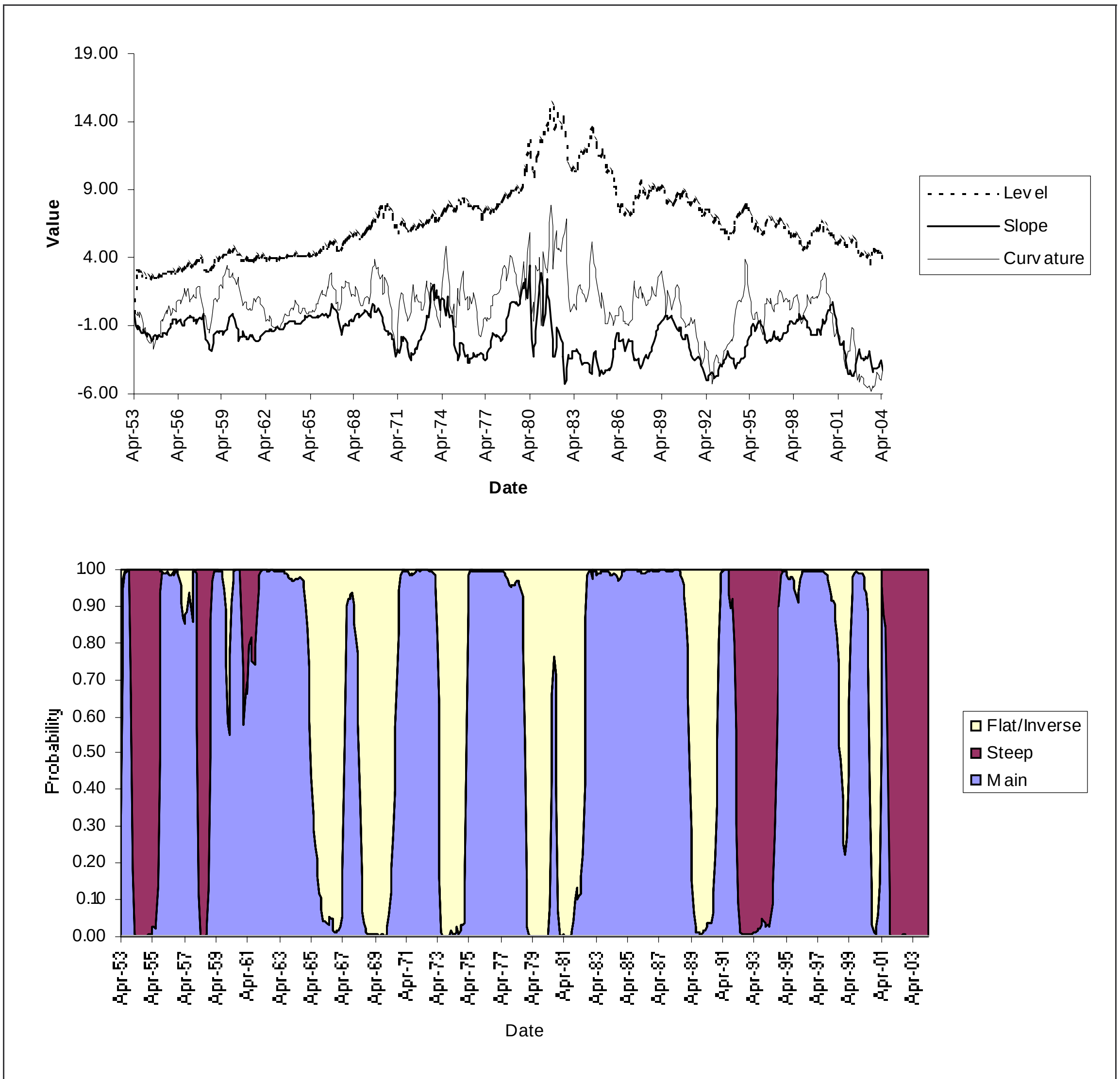
Annex 2: Figures

Figure 1: Yield curve and macroeconomic data



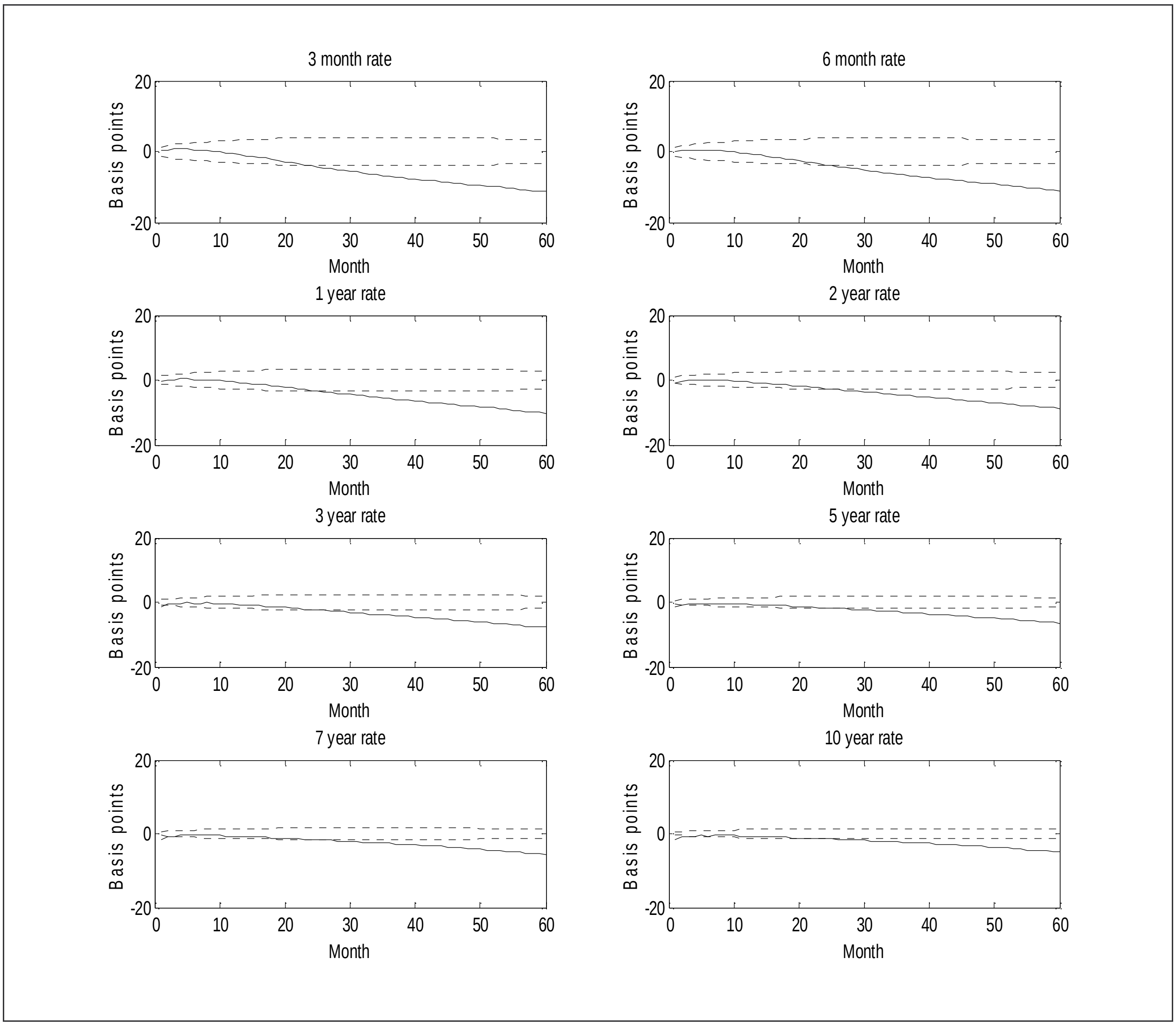
Note: The upper panel shows the time-series evolution of the data used in the study for the observed maturity segments {3months, 6months, 1year, 2year, 3year, 5year, 7year, 10year}. The lower panel shows the time-series evolution of the macro-economic variables i.e. GDP and CPI growth.

Figure 2: Estimates yield curve factors, regime classifications and generic yield curves



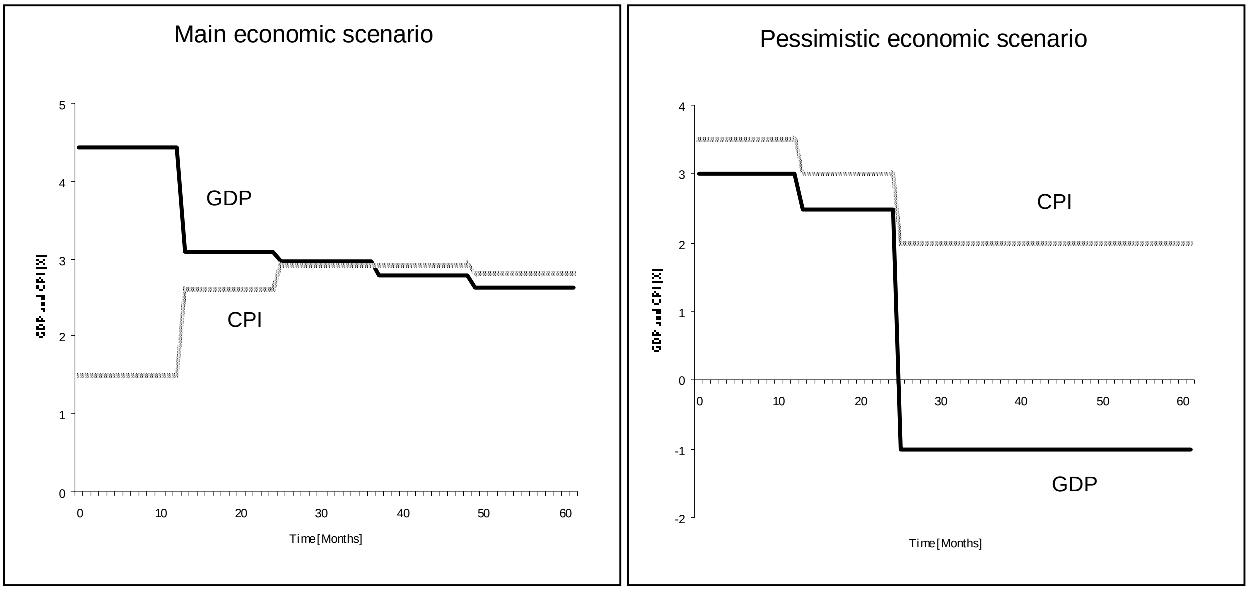
Note: The upper panel shows the evolution of the three estimated yield curve factors. Estimations are performed on scaled data following [7], but results in the upper panel have been rescaled to enhance readability. The lower panel shows the obtained regime classifications and hence plots $\pi_{t|t}$ from [9].

Figure 3: Forecast performance, comparing rmse of the Regime Switching model to the rmse of the Diebold&Li model.



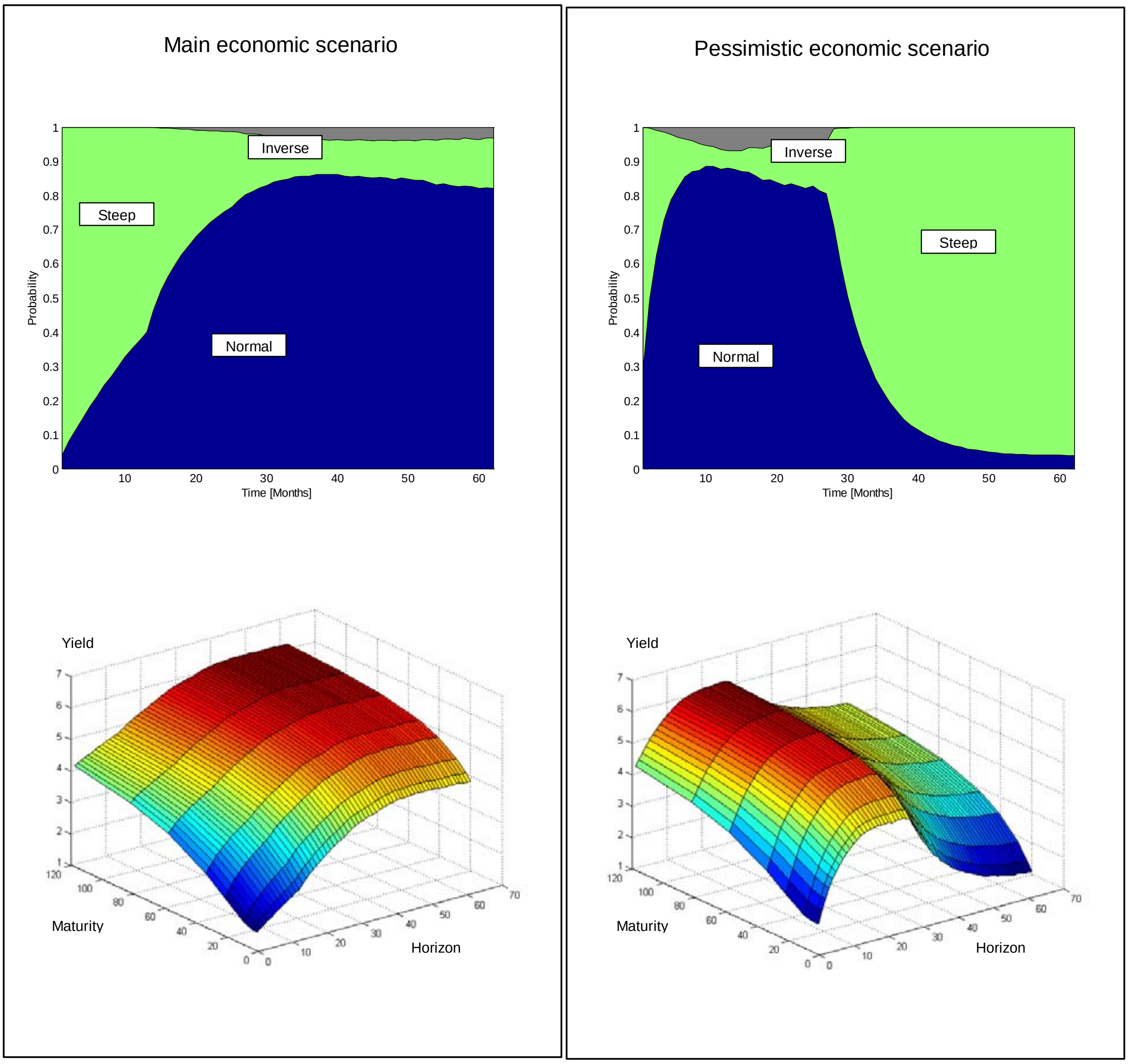
Note: Each sub-plot shows the difference between the root mean squared error (rmse) of the regime-switching model and the rmse of the Diebold and Li method as well as 95% confidence limits, for the different yield curve segments included in the original data sample. The full-lines depict $rmse_{RS} - rmse_{DL}$, so a negative number in the graph signifies better forecasting performance of the regime-switching model. The dotted lines are 95% upper and lower confidence intervals calculated on the basis of Diebold and Mariano (1995).

Figure 4: Distribution of GDP and inflation over the forecast horizon



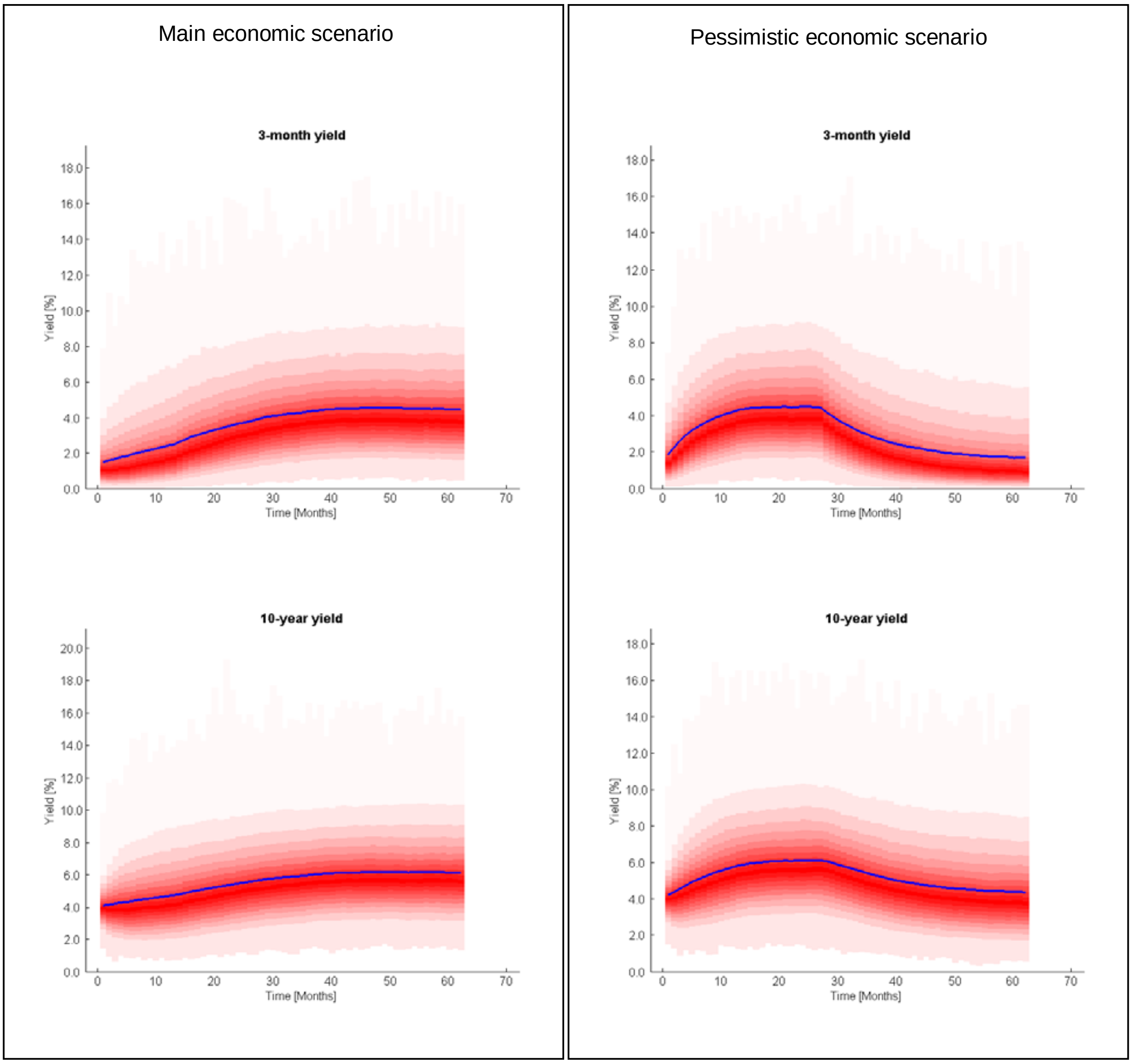
Note: Two hypothetical macro economic scenarios are show for the evolution of GDP and CPI growth. The forecasting horizon is 60 months and the scenarios represent a main economic evolution (in the left most panel) and a pessimistic macro scenario (in the right most panel).

Figure 5: Evolution of average state probabilities and yield curve



Note: The left panel shows the evolution of the regime probabilities over the forecasting horizon and the resulting projected yield curve surface, for a hypothetical main economic scenario. The right panel shows the evolution of regime probabilities and the yield curve surface when the hypothetical macro scenarios is based on a pessimistic economic scenario. The hypothetical evolutions correspond to the example projections for GDP and CPI growth depicted in Figure 4.

Figure 6: Distribution of 3 months and 10 years yields



Note: The left-most panel shows the distribution of the 3 months and 10 year segments of the yield curve across the 10,000 conducted simulations as described in Annex 1 for a hypothetical main economic scenario. The right-most panel shows the distribution of the 3 months and 10 year segments of the yield curve across the 10,000 conducted simulations as described in Annex 1 for a hypothetical pessimistic economic scenario.

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